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LETTER TO THE EDITOR

Some comments on "An Asymptotic Theory For Dynamic Response of Doubly Curved Laminated Shells", *International Journal of Solids and Structures*, Vol. 33, No. 26, pp. 3813–3841 (1996) by Chih-Ping Wu, Jiann-Quo Tarn and Shu-Man Chi.

The present author has noticed with particular interest the very recent paper by Wu *et al.* (1996), which presented a very successful asymptotic analysis suitable for the study of the linear dynamic response of doubly curved cross-ply laminates. This author would initially like to congratulate Wu *et al.* (1996) for the excellent piece of mathematical work they presented. Moreover, he would like to briefly express some views regarding the appearance of this new asymptotic analysis (Wu *et al.*, 1996) within the general field of three-dimensional dynamic analysis of laminated composite shells. Finally he would like to comment with regard to some of the numerical results presented in (Wu *et al.*, 1996), and compared with corresponding results due to Bhimaraddi (1991), and to further compare them with corresponding results based on his own research publications.

As it was mentioned in the Introduction of (Wu *et al.*, 1996), "three dimensional analyses of doubly curved laminated shells are scarce" in the literature. Moreover, even three dimensional dynamic analyses of homogeneous isotropic and orthotropic doubly curved shells are scarce. Contrary to this, and in accordance with a recent review paper published by Soldatos (1994), three-dimensional dynamic analysis of homogeneous and laminated composite circular cylinders are more commonly met in the literature. There are still very many unsolved problems remaining in the area of three-dimensional dynamic analysis of homogeneous and composite circular cylinders. However, numerical results that are based on the already solved relevant problems can be used as benchmarks for the validity of corresponding dynamic analyses of doubly curved shells. In this respect, the benchmark results chosen by Wu *et al.* (1996) were based on Bhimaraddi's (1991) successive approximation analysis. The reason behind it might be the fact that Bhimaraddi's (1991) analysis had also been applied for the dynamic analysis of composite spherical shells.

Bhimaraddi (1991) and Fan and Zhang (1992) successive approximation analyses, for doubly curved cross-ply laminated shells, are essentially extensions of a corresponding dynamic analysis published slightly earlier, for homogeneous isotropic circular cylindrical shells, by Soldatos and Hadjigeorgiou (1990). Wu *et al.* (1996) suggested that "this method may not be advantageous in computations because of the necessity of solving a large system of equations resulting from imposing the interfacial continuity conditions". Ye and Soldatos (1994a), however, have already shown that this is not the case. On the contrary, upon employing the transfer matrix method in connection with the method employed in (Soldatos and Hadjigeorgiou 1990), Ye and Soldatos (1994a) were able to reduce it into a refined recursive formulation which, regardless of the number of real and fictitious layers involved in a laminated cylinder, provides always the natural frequencies of vibration as the roots of a 6×6 frequency determinant.

Moreover, in a series of relevant publications (Soldatos and Hadjigeorgiou, 1990; Soldatos 1991; Ye and Soldatos, 1994a, 1994b; Soldatos and Ye, 1995), this author and his co-workers confirmed that their approach is essentially exact in the sense that it practically produces, where available, the results obtained by means of other analytical methods (e.g., the method of Frobenius). There were, however, two cases in which Ye and Soldatos (1994a) and (1994b) failed to reproduce the results obtained elsewhere, these being related

Table 1. Comparison of fundamental frequency parameters $\Omega = \omega L_x(\rho/E_2)^{1/2}$ of two-layered regular antisymmetric cross-ply laminated cylindrical panels

R/L_x		$h/L_x = 0.05$	$h/L_x = 0.10$	$h/L_x = 0.15$
1	Wu *et al.* (1996)	0.79307	1.06875	1.34070
	Ye and Soldatos (1994a)	0.79316	1.06973	1.34537
	Bhimaraddi (1991)	0.78683(0.8%)	1.04085(2.7%)	1.29099(4.0%)
5	Wu *et al.* (1996)	0.49331	0.90573	1.24210
	Ye and Soldatos (1994a)	0.49346	0.90616	1.24524
	Bhimaraddi (1991)	0.49167(0.4%)	0.90200(0.5%)	1.23849(0.5%)
10	Wu *et al.* (1996)	0.47948	0.89740	1.23413
	Ye and Soldatos (1994a)	0.47959	0.89778	1.23707
	Bhimaraddi (1991)	0.47859(0.2%)	0.89564(0.2%)	123374(0.3%)

to Bhimaraddi's (1991) dynamic analysis and its corresponding static equivalent (Bhimaraddi and Chandrashekhara, 1992), respectively.

Since the benchmark results chosen by Wu *et al.* (1996) are based on Bhimaraddi's (1991) dynamic analysis, only this case is considered in this brief discussion. In this respect, for certain simply supported circular cylindrical shells having a two-layered regular antisymmetric cross-ply lay up, the corresponding fundamental frequencies obtained by Bhimaraddi (1991), Ye and Soldatos (1994a) and Wu *et al.* (1996) are all tabulated and compared in Table 1. The material properties of the shell panels considered can be found in all three papers, while the geometrical notation employed is that used by Ye and Soldatos (1994a). Keeping the pattern adopted in (Ye and Soldatos, 1994a), enclosed in parentheses are the approximate absolute values of the relative error observed with Bhimaraddi's (1991) corresponding results. Although in some cases that consider very short (and therefore shallow) or very thin shells this error is relatively small, it becomes as high as 4% for relatively longer ($R/L_x = 1$) and thick ($h/L_x = 0.15$) cylinders. Contrary to this, there is always an excellent agreement between corresponding results due to Wu *et al.* (1996) and Ye and Soldatos (1994a), as the results always agree to up at least three significant figures.

The comparisons performed in Table 1 show that the two different approaches employed in (Ye and Soldatos, 1994a) and (Wu *et al.*, 1996) produce practically identical numerical results. Some very slight disagreements observed after the third significant figure of the natural frequencies calculated, cannot be considered as a drawback for either of the two methods. Contrary to this, it should be concluded that (like in Frobenius method) both methods are capable of producing the exact frequencies of vibration, provided that an adequately large number of terms will be retained in Wu *et al.* (1996) asymptotic analysis and an adequately large number of iterations will be performed through Soldatos and Ye (1994a) recursive formulation.

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REFERENCES

Bhimaraddi, A. (1991) Free vibration analysis of doubly curved shallow shells on rectangular planform using three-dimensional elasticity theory. *International Journal of Solids and Structures* **27**, 897–913.

Bhimaraddi, A. and Chandrashekhara, K. (1992) Three-dimensional elasticity solution for static response of simply supported orthotropic cylindrical shells. *Composite Structures* **20**, 227–235.

Fan, J. and Zhang, J. (1992) Analytical solutions for thick, doubly curved, laminated shells. *ASCE Journal of Engineering Mechanics* **118**, 1338–1356.

Soldatos, K. P. (1991) An iterative solution of a Bessel's equation based on torsional vibrations of orthotropic hollow cylinders. *Journal of Sound and Vibration* **151**, 149–152.

Soldatos, K. P. (1994) Review of three dimensional dynamic analyses of circular cylinders and cylindrical shells. *Applied Mechanics Reviews* **47**, 501–516.

Soldatos, K. P. and Hadjigeorgiou, V. P. (1990) Three-dimensional solution of the free vibration problem of homogeneous isotropic cylindrical shells and panels. *Journal of Sound and Vibration* **137**, 369–384.

Soldatos, K. P. and Ye, J. Q. (1995) Axisymmetric static and dynamic analysis of laminated hollow cylinders composed of monoclinic elastic layers. *Journal of Sound and Vibration* **184,** 245–259.

Wu, C.-P., Tarn, J.-Q. and Chi, S.-M. (1996) An asymptotic theory for dynamic response of doubly curved laminated shells. *International Journal of Solids and Structures* **33,** 3813–3841.

Ye, J. Q. and Soldatos, K. P. (1994a) Three-dimensional vibrations of laminated cylinders and cylindrical panels with a symmetric or an antisymmetric cross-ply lay-up. *Composite Engineering* **4,** 429–444.

Ye, J. Q. and Soldatos, K. P. (1994b) Three-dimensional stress analysis of orthotropic and cross-ply laminated hollow cylinders and cylindrical panels. *Computer Methods in Applied Mechanics and Engineering* **117,** 331–351.